Workshop on CSR in Storage Rings
Napa, October 27-28, 2002

CSR and Longitudinal Beam Dynamics: a Numerical Study

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(*with R. Warnock and R. Ruth*)

Outline

- 1D Vlasov-Fokker-Planck equation to study interplay between CSR and its dynamical effects.
- Simplified model for CSR impedance (parallel plates)
- Solve VFP equation numerically.
- Two regimes:
  - » Bursting **(NSLS VUV Storage Ring)**
  - » Steady state emissions **(Bessy II).**

In Vlasov (Vino?) Veritas

VFP Equation

$$q = z/\sigma_z, \qquad p = -\Delta E/\sigma_E$$

$$\frac{\partial f}{\partial \tau} + p\frac{\partial f}{\partial q} - [q + F_c(q,\tau,f)]\frac{\partial f}{\partial p} = \frac{2}{\omega_s t_d}\frac{\partial}{\partial p}\left(pf + \frac{\partial f}{\partial p}\right)$$

$\tau = \omega_s t$

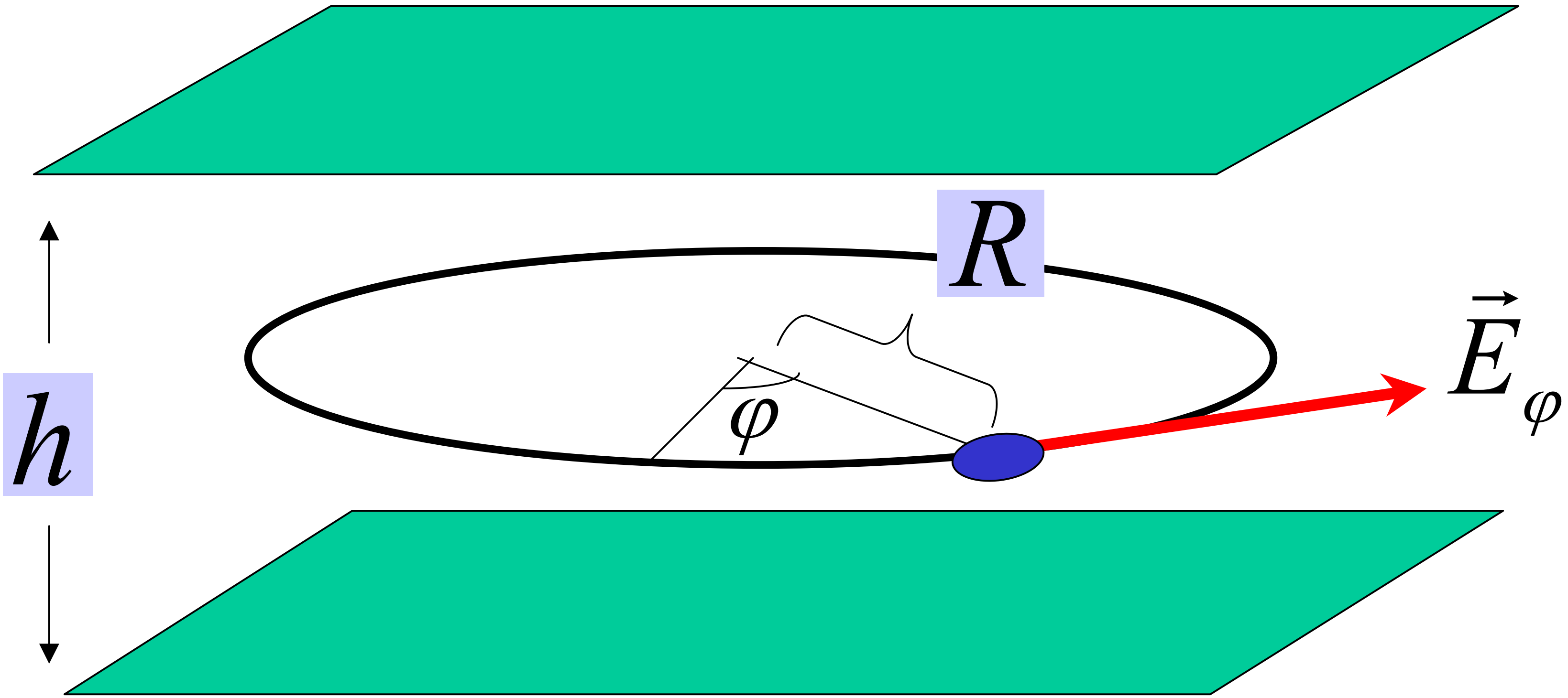
RF focusing

Collective Force

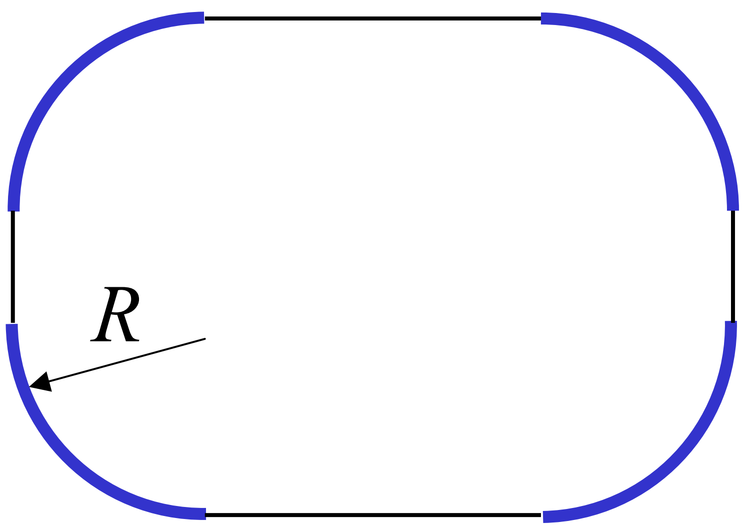
Damping

Quantum Excitations

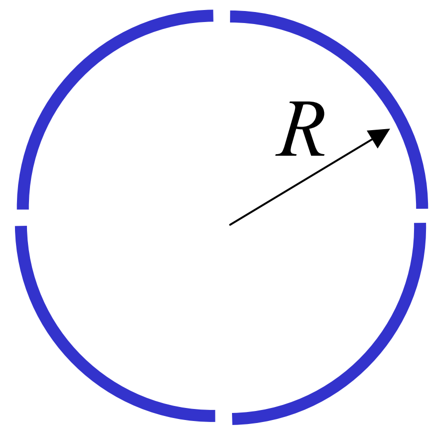
Parallel Plate Model for CSR: Geometry Outline



Actual lattice



Simplified lattice for CSR force calculation



$$2\pi\langle R\rangle E_{\varphi}^{\langle R\rangle} = 2\pi\, R\, E_{\varphi}^{R}$$

Collective Force due CSR
(also, see R. Warnock talk)

FT of (normalized) charge density of bunch.

$$E_\varphi = -\frac{e\omega_0 N}{(2\pi)^2 R}\sum_n e^{in\varphi}\int_{-\infty}^{\infty} d\omega e^{-i\omega t} Z(n,\omega)\int_{-\infty}^{t} dt' e^{i(\omega - n\omega_0)t'}\lambda_n(t')$$

Assume charge distribution doesn't change much over one turn (rigid bunch approx).

$$E_\varphi = -\frac{e\omega_0 N}{2\pi R}\sum_n e^{in(\varphi - n\omega_0 t)} Z(n, n\omega_0)\lambda_n(t')$$

…more on collective force

$$\frac{\partial f}{\partial \tau} + p\frac{\partial f}{\partial q} - [q + F_c(q,\tau,f)]\frac{\partial f}{\partial p} = \frac{2}{\omega_s t_d}\frac{\partial}{\partial p}\left(pf + \frac{\partial f}{\partial p}\right)$$

$$F_c(q,\tau,f) = -I_c\omega_0\sum_n e^{inq\sigma_z/R}Z(n)\lambda_n$$

$$I_c = \frac{e^2 N}{2\pi\nu_s\sigma_E} = \frac{e^2 N\sigma_z}{2\pi\alpha\langle R\rangle E_0\sigma_\delta^2} \qquad \left[\frac{C}{V}\right]$$

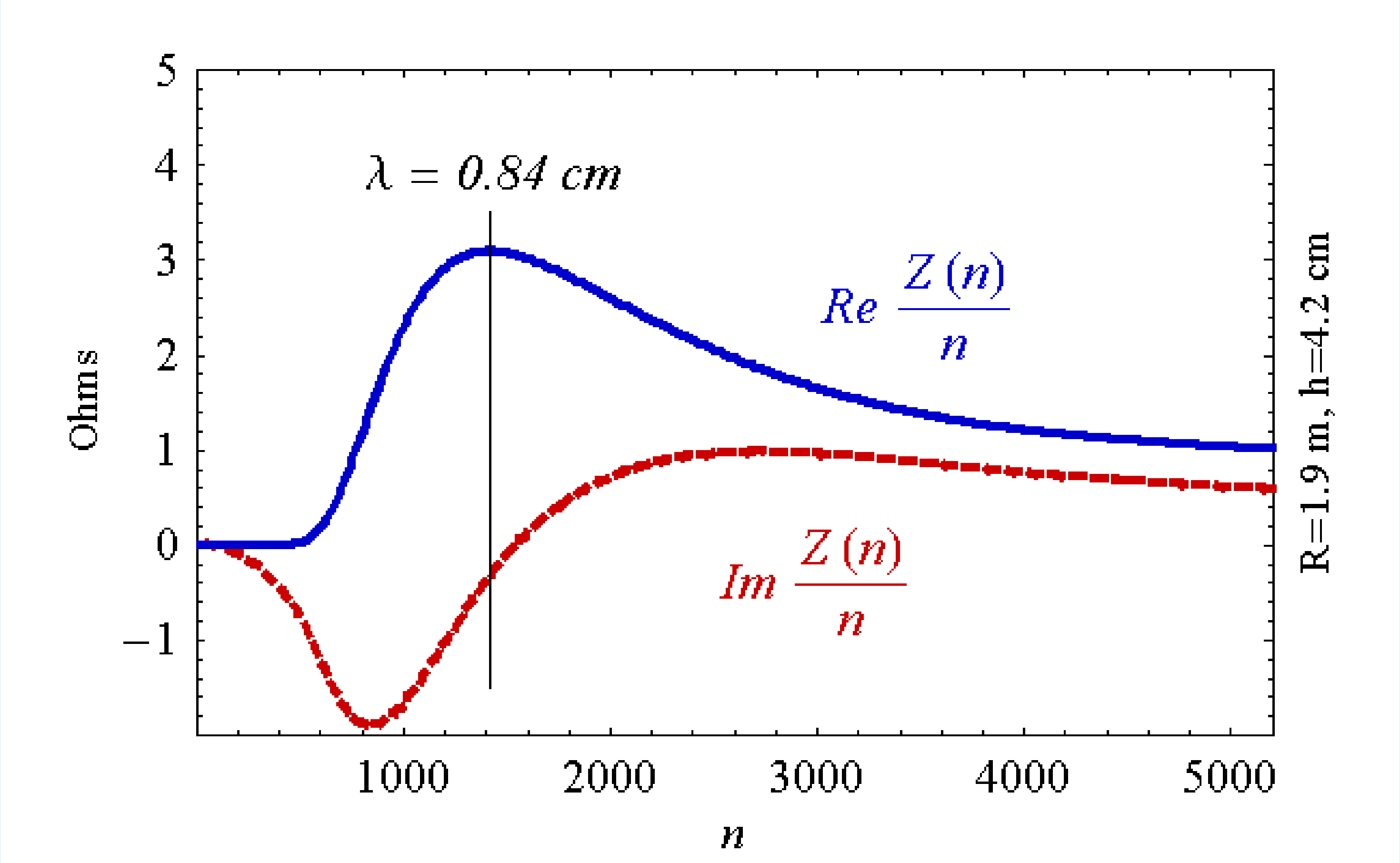
NSLS VUV Ring Parameters

NSLS
NATIONAL SYNCHROTRON LIGHT SOURCE

Energy	737 MeV
Average machine radius	8.1 m
Local radius of curvature	1.9 m
Vacuum chamber **aperture**	4.2 cm
Nominal bunch length (rms)	5 cm
Nominal energy spread (rms)	$\sigma_\delta = 5 \times 10^{-4}$
Synchrotron tune	$\nu_s = 0.018$
Longitudinal damping time	10 ms

Parallel Plate Model : NSLS VUV Ring

$$Z(n) \equiv Z(n, n\omega_0)$$



Analytic Expression for CSR Impedance (Parallel Plates)

By definition: $-2\pi R E_\varphi(n,\omega) = Z(n,\omega) I_\varphi(n,\omega)$

Impedance
$$\frac{Z(n,\omega)}{n} = \frac{2\pi^2 Z_0}{\beta}\frac{R}{h}\sum_{p=1,3,\ldots}\Lambda_p\left[\frac{\beta\omega R}{nc}J'_n H'^{(1)}_n + \frac{\alpha_p^2}{\gamma_p^2}J_n H^{(1)}_n\right]$$

With $\alpha_p = \pi p / h$, $\gamma_p^2 = (\omega / c)^2 - \alpha_p^2$

Argument of Bessel functions $R\gamma_p$

$\Lambda_p = \sin(x) / x, x = \pi p \delta h / 2h$

δh = beam height

Divide and Conquer

Numerical Method

- Solve VFP by operator splitting (propagate distribution under Vlasov and Fokker-Planck parts separately).
- Propagate distribution function under FP by a simple Euler step using finite differences (no need to be fancy here).

Propagation of Distribution under Vlasov

- Idea: follow the particle trajectories (method known as Semi-Lagrangian, Perron-Frobenius, Time-Splitting …)

- Implemented with the Yabe *et al*. variation: propagate derivatives $\partial_q f$, $\partial_q f$ alongside with f

Perron-Frobenius Operator

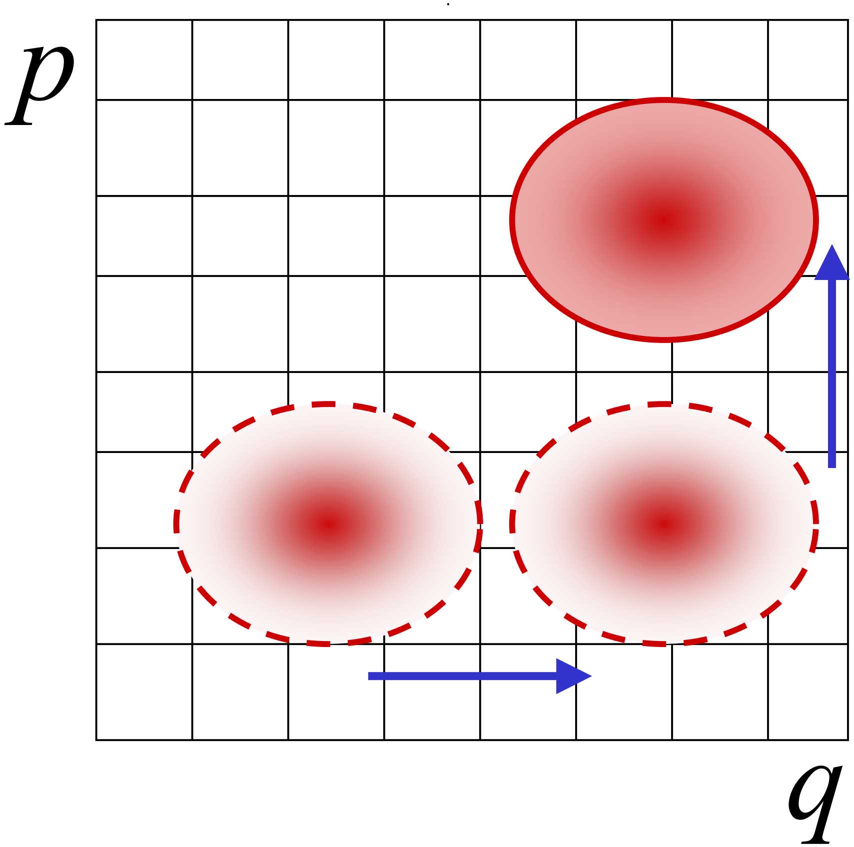
- $z = (q, p)$ is point in phase-space.

- Mapping induced by the Hamiltonian dynamics from time t_0 to time t :

$$z \to z' = M_{t_0 \to t}(z)$$

- Propagation of f (Perron-Frobenius operator)

$$f(z,t) = f(M^{-1}_{t_0 \to t}(z), t_0)$$



- Do one time step along q, and one time step along p.

- Discretize in time; represent f on grid

$$f(z_{ij}, t+\Delta t) = f(M^{-1}_{t\to\Delta t}(z_{ij}), t_0)$$

- Break map into sequence of kicks

$$M_{t\ \to t+\Delta t} \approx M^{(p)}_{t\ \to t+\Delta t} \circ M^{(q)}_{t\ \to t+\Delta t}$$

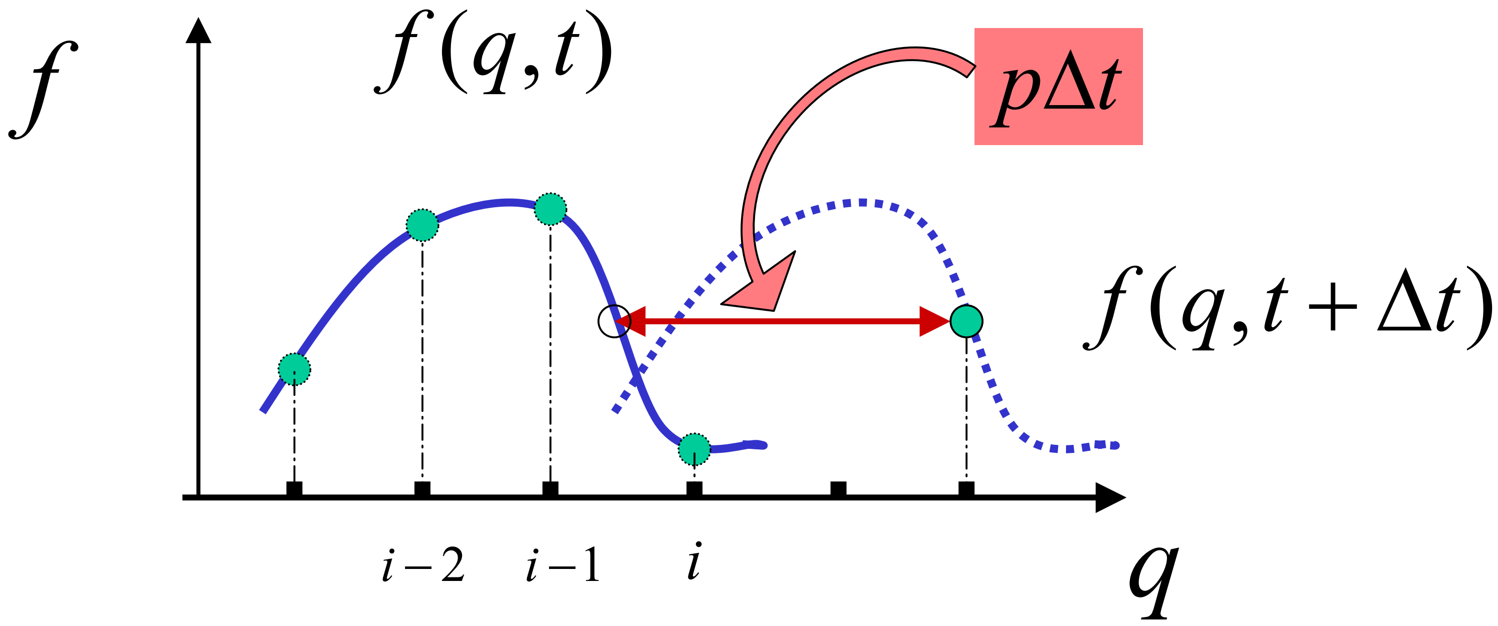
$$M^{(q)}_{t\to t+\Delta t}: \begin{cases} p' = p \\ q' = q + p\Delta t \end{cases}$$

$$M^{(p)}_{t\to t+\Delta t}: \begin{cases} p' = p - [q + F_c(q, f)]\Delta t \\ q' = q \end{cases}$$

Step along q (pure drift)

$$f(q, t+\Delta t) = f(q - p\Delta t, t)$$

Do cubic interpolation between grid-points
Using f and $\partial_q f$ at end-points.



Propagation of Derivatives

$$\partial_q f(q, t+\Delta t) = \partial_q f(q - p\Delta t, t)$$

- Propagate $\partial_q f$ by differentiating same interpolating polynomial used to propagate f

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- For use finite differences. Start from Vlasov:

$$\partial_t f = p\partial_q f \quad \overset{\partial_p}{\Longrightarrow} \quad \partial_t(\partial_p f) = \partial_p(p\partial_q f)$$

$$\partial_p f(t+\Delta t) = \partial_p f(t) + \Delta t\left(\frac{(p\partial_q f)_{j+1} - (p\partial_q f)_{j-1}}{2\Delta p}\right)$$

In Boussard We Trust.

Stability Analysis of Linear Motion

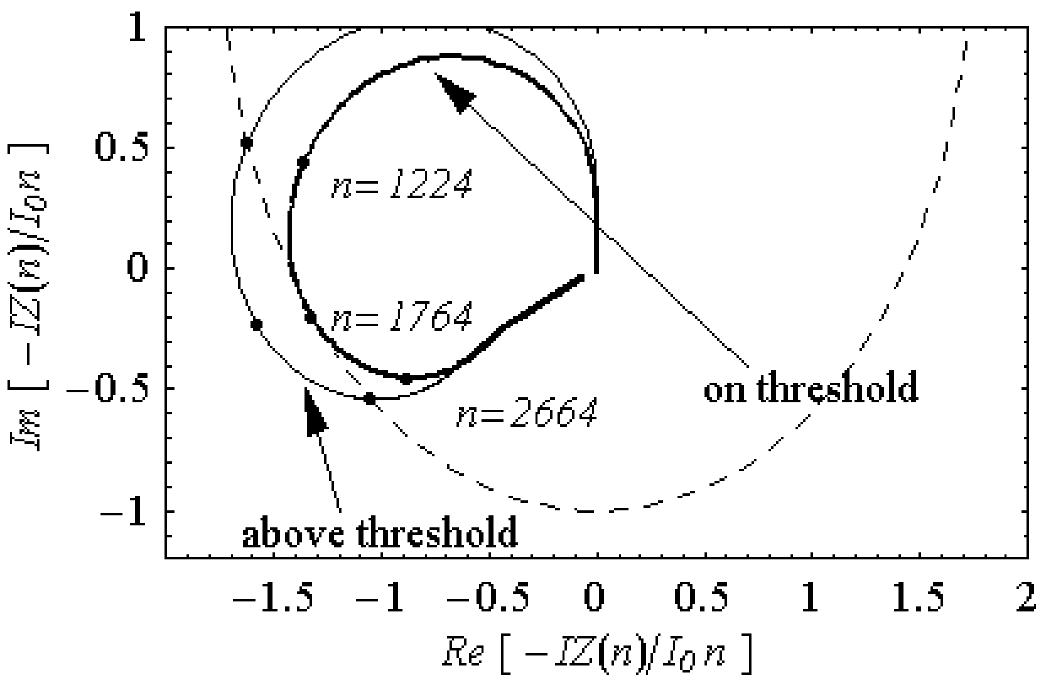
- Boussard criterion (use equivalent coasting beam with same peak current)

- Gaussian distribution in energy spread

- Dispersion relation $$\frac{I_c \omega_0 R^2}{\sigma_z^2 \sqrt{2\pi}} \frac{Z(n)}{n} = \frac{i}{W\left(\nu R / \sigma_z n\right)}$$

Error function of complex arg

Keil-Schnell Diagram



Most unstable harmonic:
$n = 1764 \Leftrightarrow \lambda = 6.7\,mm$

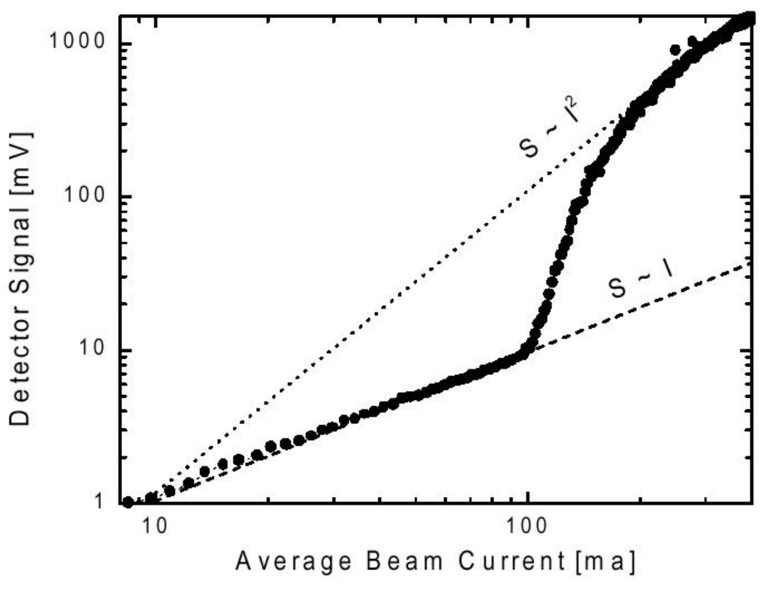
	Measurements*	Boussard	Solver
Current threshold	100 mA $I_c = 3.7\ pC/V$ $N = 1.1\times 10^{11}$	$I_c = 6.2\ pC/V$ $N = 1.8\times 10^{11}$	$I_c = 6.6\ pC/V$ $N = 1.9\times 10^{11}$
CSR wavelength	7 mm	6.7 mm	7.8 mm

* G. Carr *et al*., NIM-A 463 (2001) p. 387

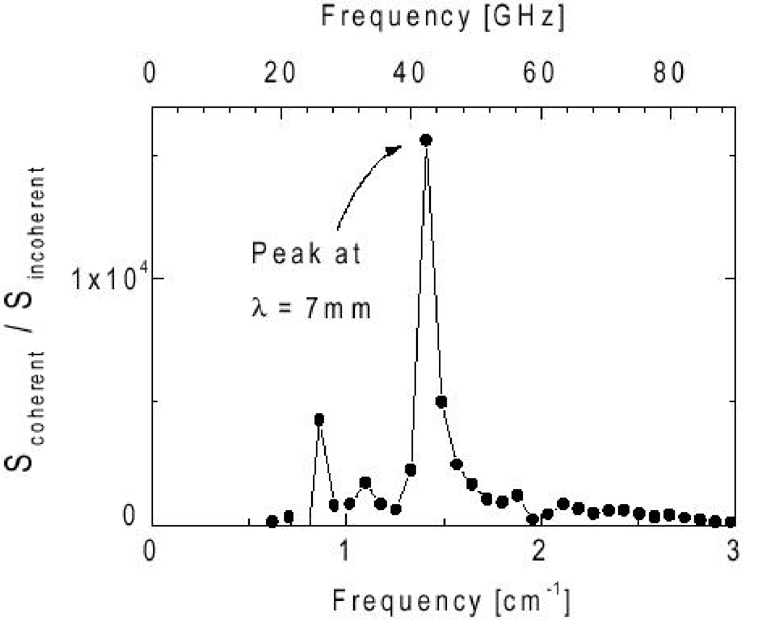
Observations of CSR - NSLS VUV Ring

Carr *et al*. NIM-A **463** (2001) p. 387

Current Threshold for Detection of Coherent Signal

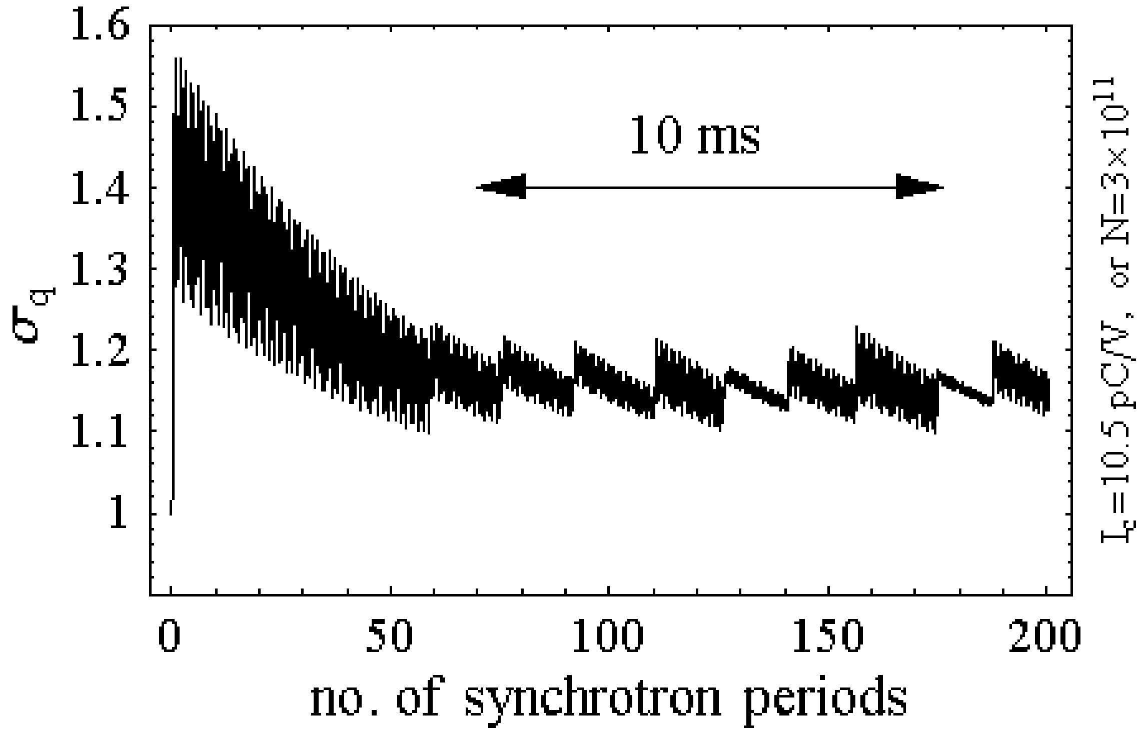


Spectrum of CSR Signal (wavelength ~ 7 mm)



It Bursts!

Bunch Length

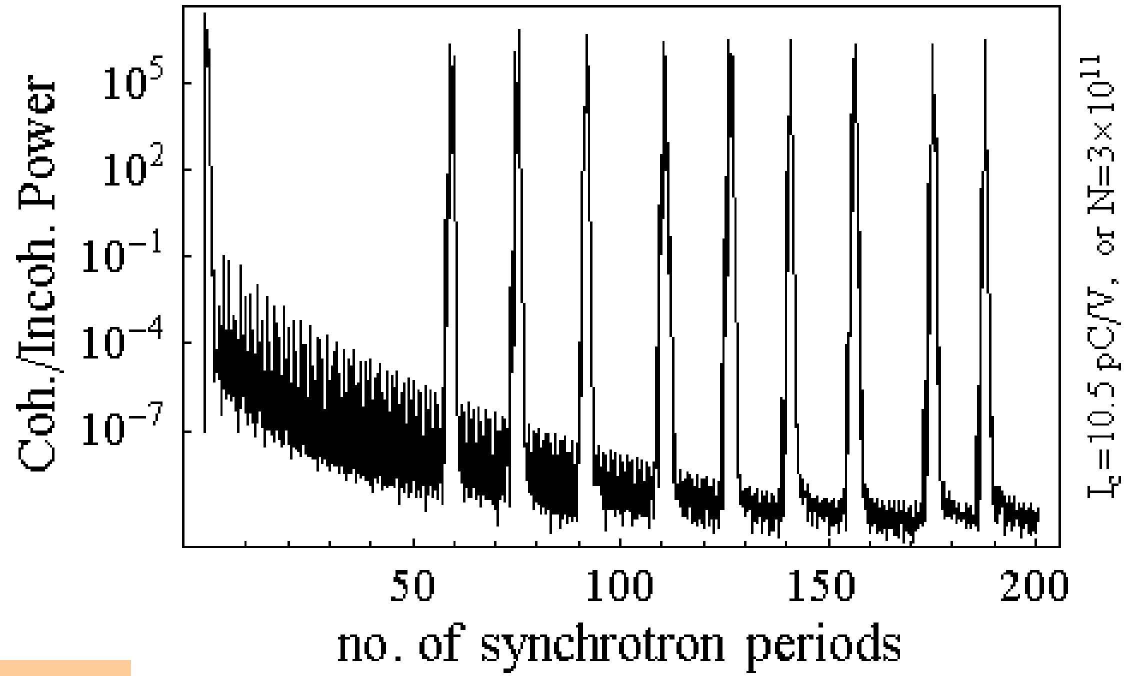


$$N = 3\times 10^{11} \equiv I_c = 10.5\, pC/V$$

$$P_n^{coh} = 2(eN\omega_0)^2 \operatorname{Re} Z(n) |\lambda_n|^2$$

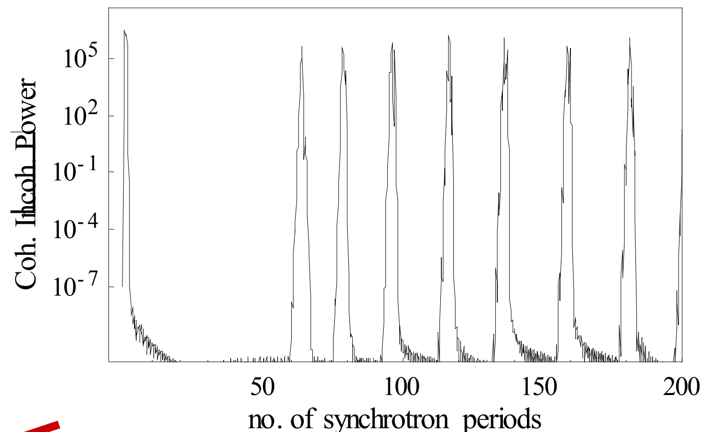
$$P_n^{incoh} = 2N(e\omega_0)^2 \operatorname{Re} Z(n) / (2\pi)^2$$

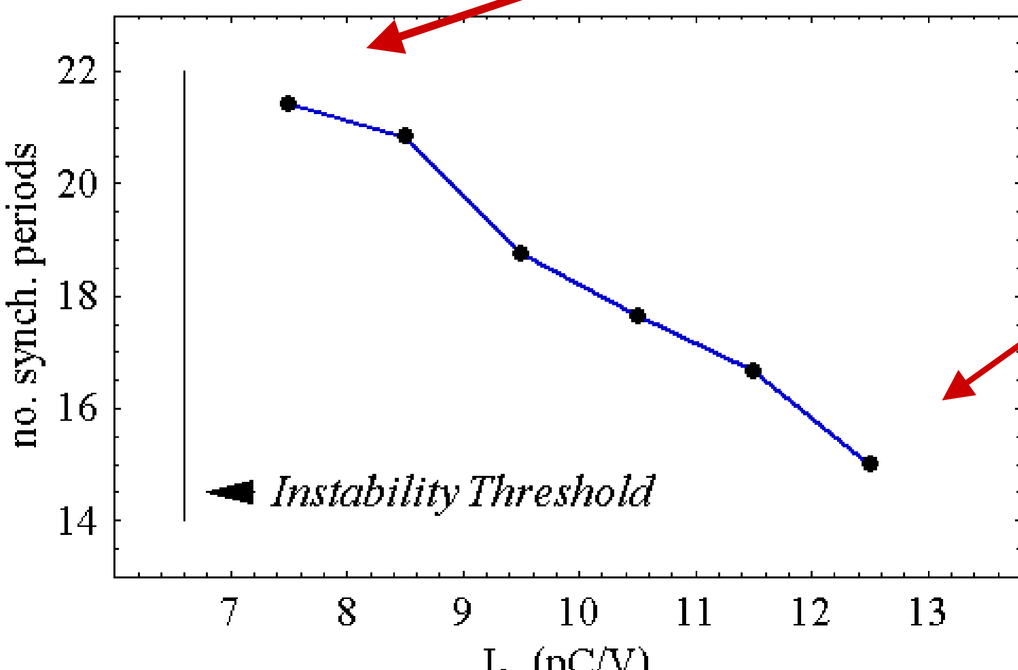
Coherent/Incoherent Power

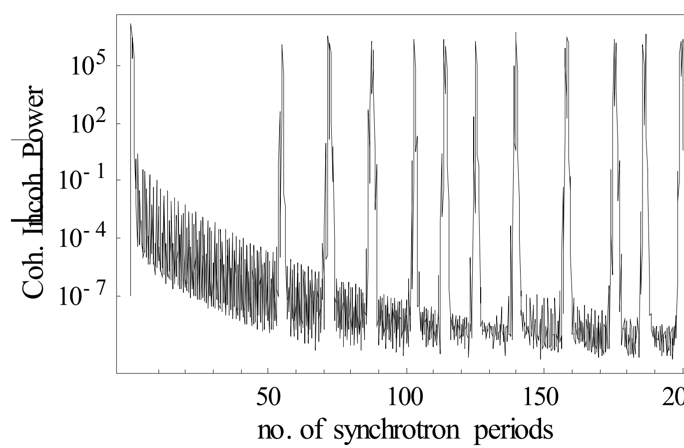


$$N = 3\times 10^{11} \equiv I_c = 10.5\ pC/V$$

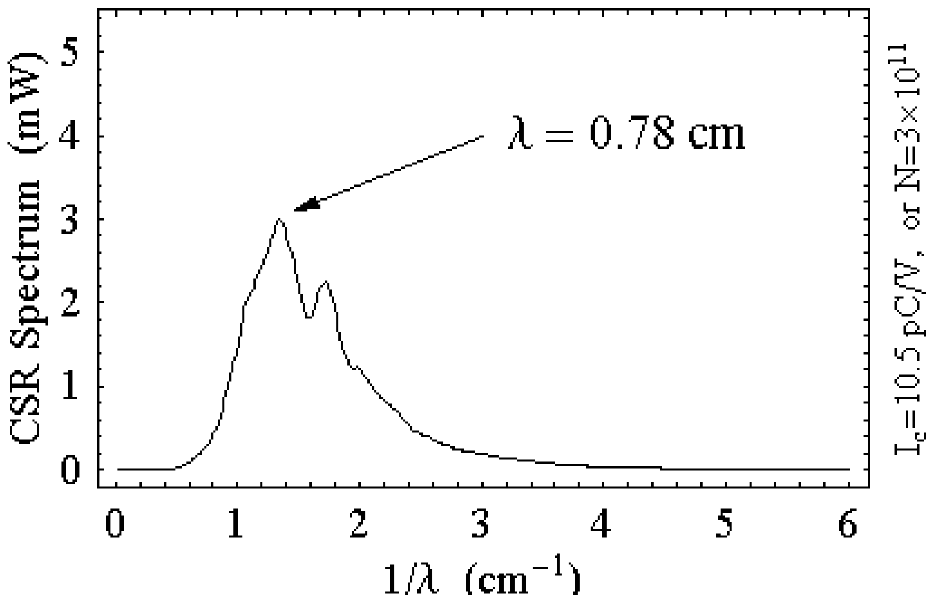
Bursts Separation vs. Current



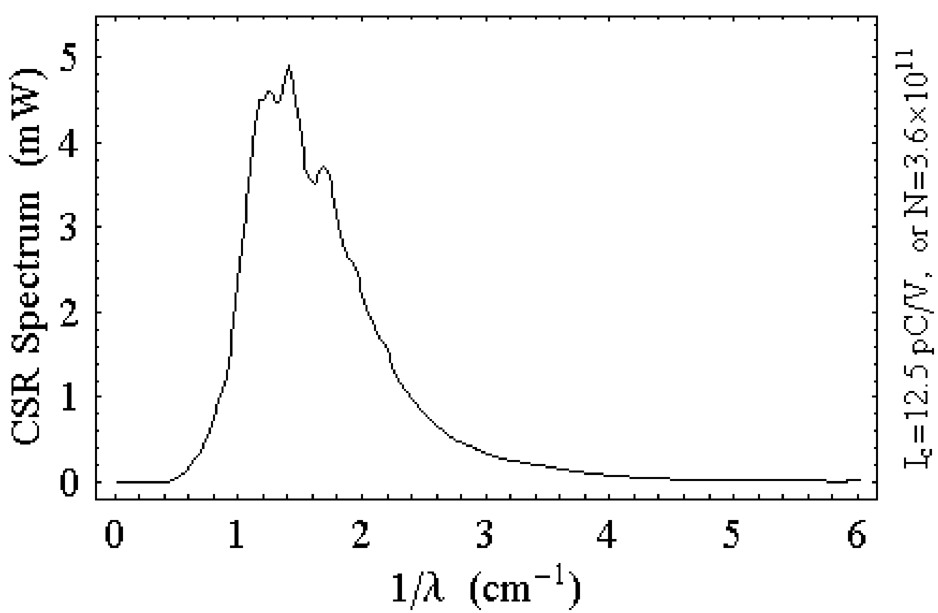




Averaged CSR Emission



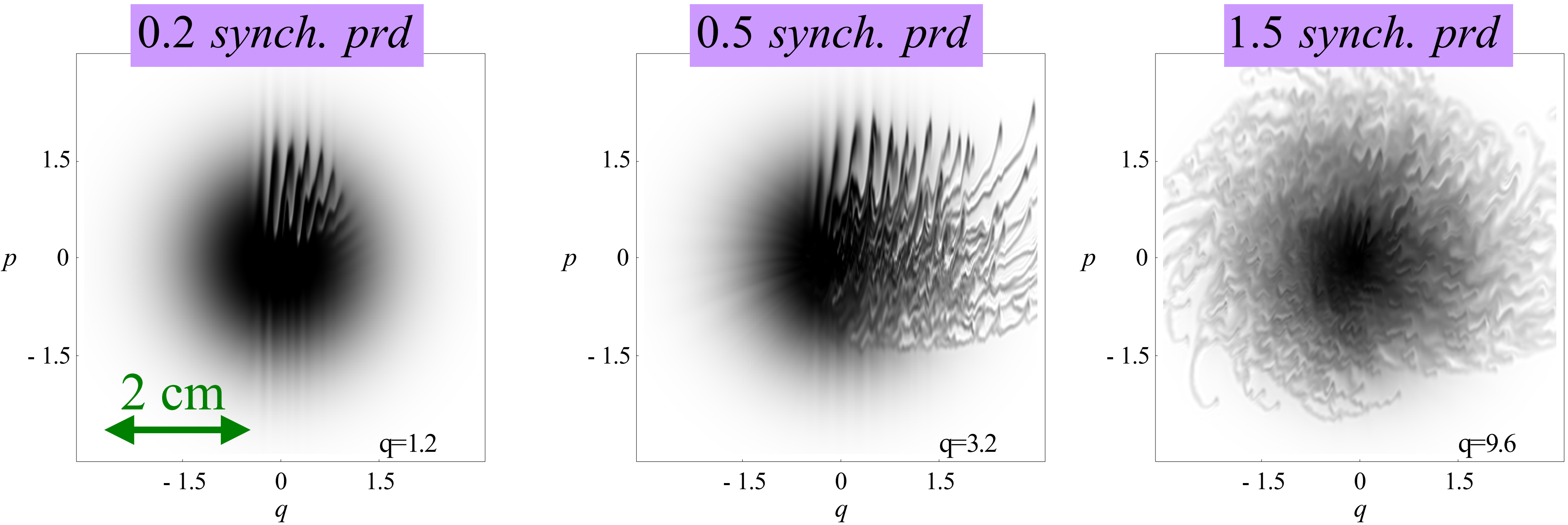
$$N = 3\times10^{11}$$



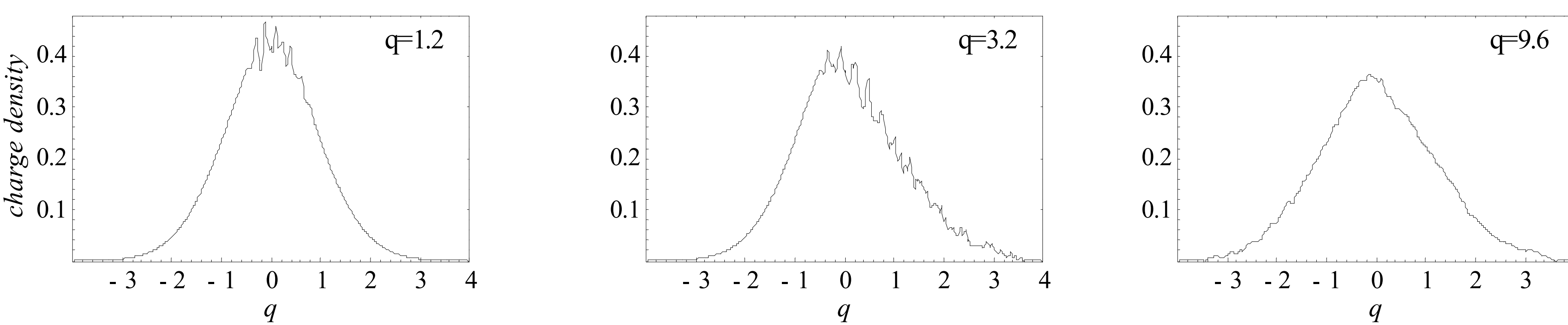
$$N = 3.6\times10^{11}$$

No damping, 25% > threshold

Density Plots in Phase space



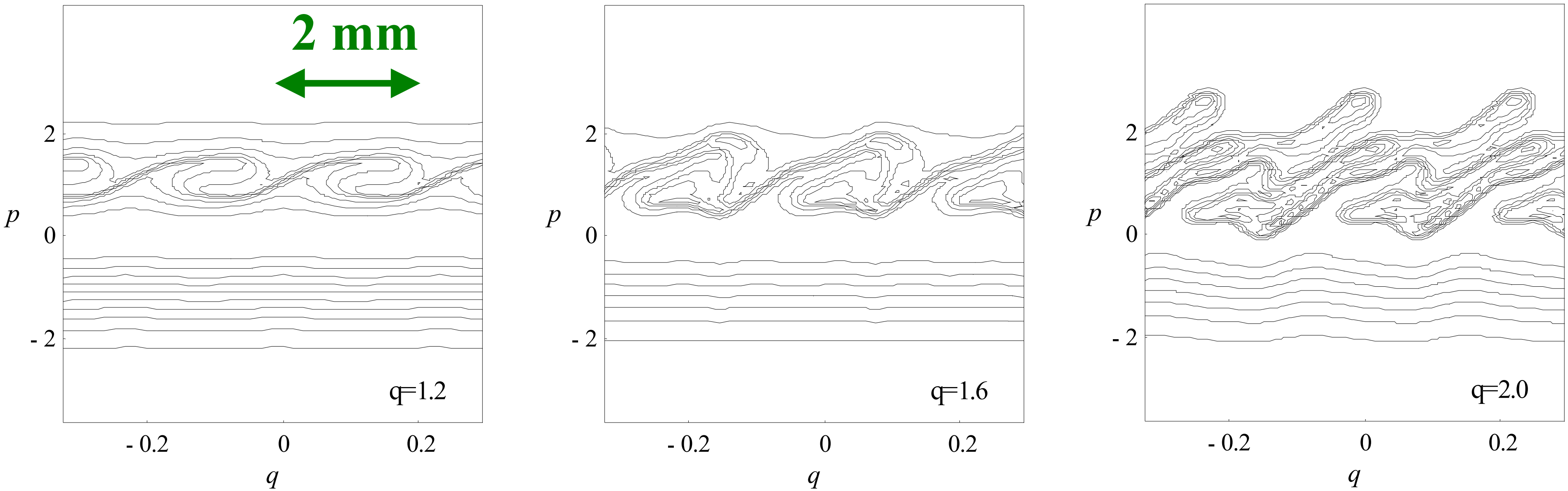
Charge Distribution



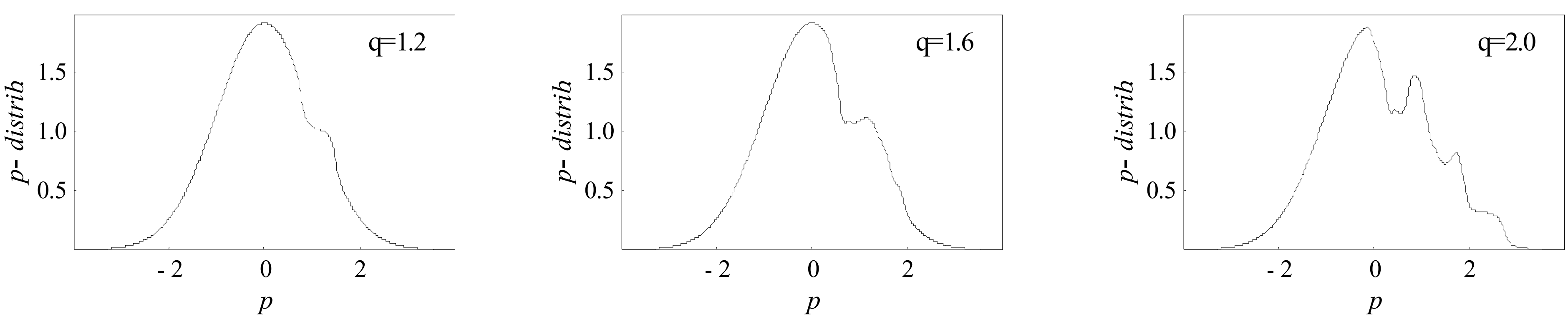
Coasting Beam

No damping, 25% > threshold

Density Contours in Phase space

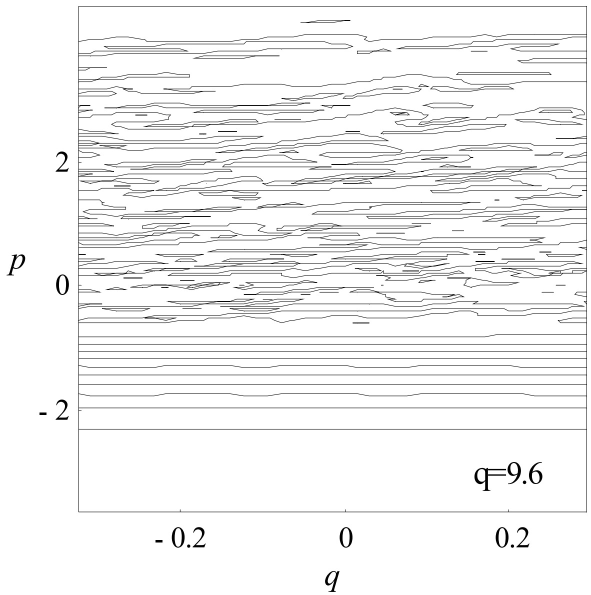


Energy Spread Distribution

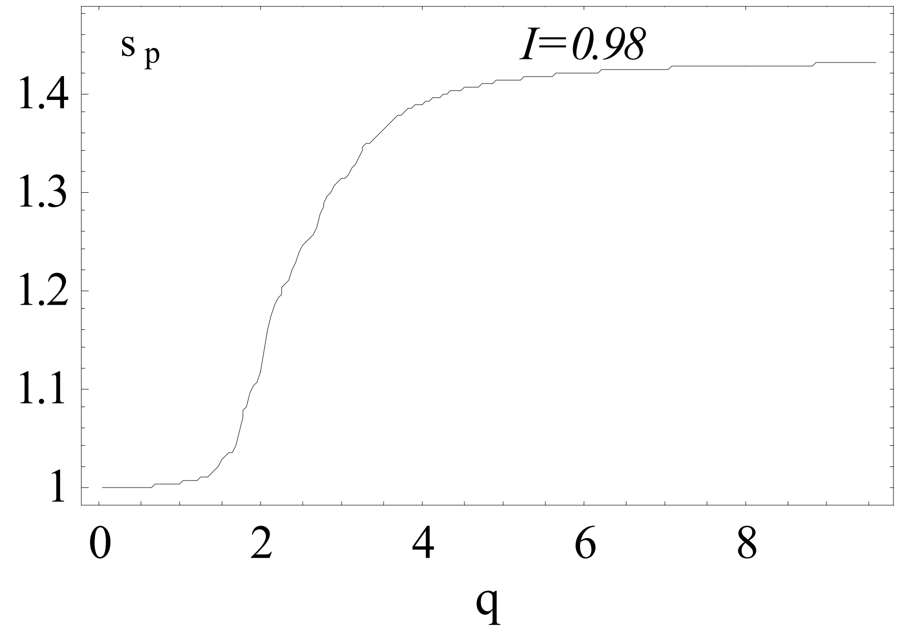


Coasting Beam: Asymptotic Solution

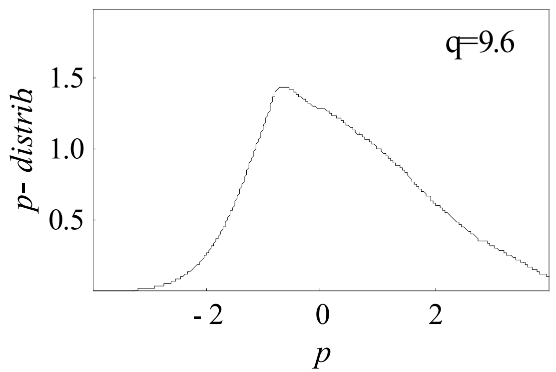
Density Contours in Phase space



Energy Spread vs. Time



Energy Spread Distribution



- Large scale structures have disappeared.
- Distribution approaches some kind of steady state.

Conclusions

- Numerical model gives results consistent with linear theory, when this applies.
- CSR instability saturates quickly.
- Saturation removes microbunching, enlarges bunch distribution in phase space.
- Relaxation due to radiation damping gradually restores conditions for CSR instability.
- CSR instability + radiation damping = sawtooth-like pattern & bursting (qualitative consistency with observations).